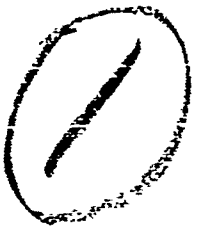


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ANALYTICAL APPROXIMATIONS

Volume 3

Cecil Hastings, Jr.

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### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho d\rho$$

in which  $I_0(z)$  is the usual Bessel function. To better than .00014 over  $(-\infty, \infty)$ ,

$$q(1, x) \doteq 1 - \frac{.3935}{[1 + .5968x^2 + .0047x^4 + .00028x^6]^4}$$

The parametric form used is convenient for approximating fixed-R cross-sections of the  $q(R, x)$  surface for small values of R.

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### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x) \rho d\rho$$

in which  $I_0(z)$  is the usual Bessel function. To better than .0035 over  $(-\infty, \infty)$ ,

$$q(2,x) \approx 1 - \frac{.865}{[1+.038x^2+.004x^4]^4}.$$

The parametric form used is convenient for approximating fixed-R cross-sections of the  $q(R,x)$  surface for small values of  $R$ .

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## Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(p^2+x^2)} I_0(px) p dp$$

in which  $I_0(z)$  is the usual Bessel function. To better than .001 over  $(-\infty, \infty)$ ,

$$q(2,x) \doteq 1 - \frac{.865}{[1+.0401x^2+.00309x^4+.000075x^6]^4}.$$

The parametric form used is convenient for approximating fixed- $R$  cross-sections of the  $q(R,x)$  surface for small values of  $R$ .

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## Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x) \rho d\rho$$

in which  $I_0(z)$  is the usual Bessel function. To better than .0014 over  $(-\infty, \infty)$ ,

$$q(1,x) \doteq 1 - \frac{.393}{[1+.093x^2 + .007x^4]^4}$$

The parametric form used is convenient for approximating fixed-R cross-sections of the  $q(R,x)$  surface for small values of R.

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### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x) \rho d\rho$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0009 over  $(0,\infty)$ ,

$$q(2,2+y) \doteq 1 - \frac{.397}{[1 + .236y + .066y^2 + .056y^3]^4}$$

The parametric form used is convenient for approximating fixed-R semi-cross-sections of the  $q(R,R+y)$  surface for any  $R \gg 0$  and for  $y$  ranging over  $(0,\infty)$ .

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